Berryman, J. E., and D. M. Himmelblau, "Effect of Stochastic Inputs and Parameters on Process Analysis and Design," *Ind. Eng. Chem. Process Design Develop.*, **10**, 441 (1971).
Bracken, J., and G. P. McCormick, *Selected Applications of Nonlinear Programming*, J .Wiley, New York (1968).
Briggs, D. E., and L. B. Evans, "An Optimization Technique for Discrete Value Problems—Heat Exchangers," *Chem. Eng. Progr. Symposium Ser. No. 50*, **60**, 23 (1964).
Chung, S. F., "Mathematical Model and Optimization of Drying Process for a Through-Circulation Dryer," *Can. J. Chem. Eng.*, **50**, 657 (1972).
Dibella, C. W., and W. F. Stevens, "Process Optimization by Nonlinear Programming," *Ind. Eng. Chem. Process Design Develop.*, **4**, 16 (1965).
Fan, L. T., and P. N. Mishra, "A Random Sampling Approach to Process System Design and Synthesis," paper presented at 76th National AIChE Meeting, Tulsa, Okla. (March, 1974).
Findley, M. E., "Modified One-at-a-Time Optimization," *AIChE J.*, **20**, 1154 (1974).
Friedman, P., and K. L. Pinder, "Optimization of a Simulation Model of a Chemical Plant," *Ind. Eng. Chem. Process Design Develop.*, **11**, 512 (1972).
Gaines, L. D., and J. L. Gaddy, "Process Optimization by Flow Sheet Optimization," *ibid.*, **15.1**, 206 (1976).
Gall, D. A., "A Practical Multifactor Optimization Criterion," in *Recent Advances in Optimization Techniques*, A. Lavi and T. P. Vogl, ed, Wiley, New York (1966).
Gottfried, B. S., P. R. Bruggink, and E. R. Harwood, "Chemical Process Optimization Using Penalty Functions," *Ind. Eng. Chem. Process Design Develop.*, **9**, 581 (1970).
Gould, F. J., "Nonlinear Pricing: Applications to Concave Programming," *Operations Research*, **19**, 1026 (1971).
Himsworth, F. R., "Empirical Methods of Optimization," *Trans. Instn. Chem. Engrs.*, **40**, 345 (1962).
Hovanessian, S. A., and T. M. Stout, "Optimum Fuel Allocation in Power Plants," *Trans. IEEE Power Apparatus Syst.*, **82**, 329 (1963).
Keefer, D. L., "Simpat: Self-Bounding Direct Search Method for Optimization," *Ind. Eng. Chem. Process Design Develop.*, **12**, 92 (1973*a*).
Luus, R., and T. H. I. Jaakola, "Optimization by Direct Search and Systematic Reduction of the Size of Search Region," *AIChE J.*, **19**, 760 (1973*a*).
————, "A Direct Approach to Optimization of a Complex System," *ibid.*, **19**, 645 (1973*b*).
Payne, R. E., "Alkylation—What You Should Know About This Process," *Petrol Refiner*, **37**, 316 (1958).
Rosen, J. B., and S. Suzuki, "Construction of Nonlinear Programming Test Problems," *Communications of the ACM*, **8**, 113 (1965).
Sauer, R. N., A. R. Colville, and C. W. Burwick, "Computer Points in the Way to More Profits," *Hydrocarbon Processing Petrol. Refiner*, **43**, 84 (1964).
Umeda, T., and A. Ichikawa, "A Modified Complex Method for Optimization," *Ind. Eng. Chem. Process Design Develop.*, **10**, 229 (1971).
White, W. B., S. M. Johnson, and G. B. Dantzig, "Chemical Equilibrium in Complex Mixtures," *J. Chem. Phys.*, **28**, 751 (1958).
Williams, T. J., and R. E. Otto, "A Generalized Chemical Processing Model for the Investigation of Computer Control," *AIEE Trans.*, **79**, 458 (1960).

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Relationships Between Velocity Profiles and Drag Reduction in Turbulent Fiber Suspension Flow

PETER F. W. LEE
and
GEOFFREY G. DUFFY
Department of Chemical and Materials Engineering
University of Auckland
Auckland, New Zealand

Analytical relationships between velocity profiles and flow resistance data are developed for suspensions of papermaking fibers in turbulent shear. The relationships apply to suspensions of synthetic fibers. The wall layer appears to be unaffected by the presence of fibers, and the cause of drag reduction can be attributed to the turbulent core region.

SCOPE

The turbulent flow of fiber suspensions is possibly unique. When fibers in suspension are subjected to high shear rates, and when there is insufficient volume for the fibers to move freely, they agglomerate by mechanical entanglement. Both free fibers and agglomerates are effective in damping turbulence and reducing energy dissipated by viscous shear to below that of the suspending medium flowing alone under the same conditions. The phenomenon of drag reduction in turbulent fiber suspensions has stimulated considerable interest in recent years and is of significant practical importance to the papermaking industry.

Correspondence concerning this paper should be addressed to P. F. W. Lee, now with The Institute of Paper Chemistry, Appleton, Wisconsin.

Most previous investigations of the drag reducing behavior of fiber suspensions have been limited to measurements of friction loss data because of the lack of suitable instrumentation for the determination of velocity profiles and turbulence parameters. An annular purge impact probe was developed by Mih and Parker (1967) to measure velocity profiles in the flow of macroscopic fiber suspensions. Their data and data obtained subsequently by Seely (1968) were limited to a small range of flow conditions.

The purpose of this work was to extend the range of velocity profiles and to investigate the relationships between flow resistance and velocity profiles for fully developed turbulent flow.

CONCLUSIONS AND SIGNIFICANCE

Velocity profile and flow resistance data were measured in this investigation over wide ranges of bulk velocity and fiber concentration for suspensions of papermaking fibers flowing in a 100 mm diameter pipe. Typical data were presented in the form of reduced velocity profiles on semilogarithmic coordinates. The reduced velocity profiles for fiber suspension flow were found to be approximately linear in regions of turbulent shear. The flow behavior near a pipe wall was apparently unaffected by the suspended fibers, and the cause of drag reduction was attributed to the turbulent core region.

The gross flow behavior and velocity distributions for pipe flow have been described analytically in terms of a single suspension parameter, the apparent von Kármán constant *K*. For a particular suspension under conditions of high intensity turbulence, *K* was found to be independent of bulk velocity and was the singularly important parameter characterizing flow. The analytical expressions developed for papermaking fibers were applied successfully to data available for synthetic fibers.

The ability of certain particulate additives to reduce turbulent flow resistance has been reported in numerous publications (Kerekes, 1970; Radkin et al., 1975). It has been established that the effectiveness of particulates as drag reducing additives increases as their aspect ratio increases and is appreciable only for fibrous additives. However, little is known about the mechanisms of drag reduction in turbulent fiber suspensions because of the tendency of fibers to interfere with a probe inserted in the flow. Vaseleski and Metzner (1974) measured flow resistance data for fiber suspensions in pipes with different diameters and inferred that the presence of fibers in the turbulent core region of flow was important for drag reduction. This is in contrast to drag reducing polymer solutions, in which the mechanism of drag reduction occurs near the wall (Virk, 1975). Further evidence for the mechanistic differences between drag reduction produced by polymeric and particulate additives was provided by Lee et al. (1974) and Kale and Metzner (1974) who used both types of additive simultaneously to achieve greater reductions in drag than when either type of additive was used separately.

It is possible to measure local velocities in the turbulent core region of fiber suspension flow by using an annular purge impact probe (Mih and Parker, 1967). In this investigation (Lee and Duffy, 1976*b*) an annular purge impact probe was used to measure velocity profiles for the turbulent flow of aqueous suspensions of papermaking fibers (average fiber length of 2.7 mm, average fiber diameter of 0.03 mm) in a 100 mm diameter hydraulically smooth pipe at bulk velocities up to 9.17 m/s.

VELOCITY PROFILES IN THE TURBULENT CORE

Typical data for local mean velocities u in suspensions with fiber concentrations of 0.21 and 0.79% are presented in Figure 1 as a graph of reduced velocity U^+ [defined by Equation (1)] vs. the logarithm of dimensionless distance S^+ [defined by Equation (2)]:

$$U^+ = \frac{u}{U^*} \tag{1}$$

where

$U^* = \sqrt{\tau_w/\rho}$
τ_w = the wall shear stress
ρ = the density of the suspending medium

$$S^+ = \frac{yU^*}{\nu} \tag{2}$$

where

y = the distance from the wall
ν = the kinematic viscosity of the suspending medium

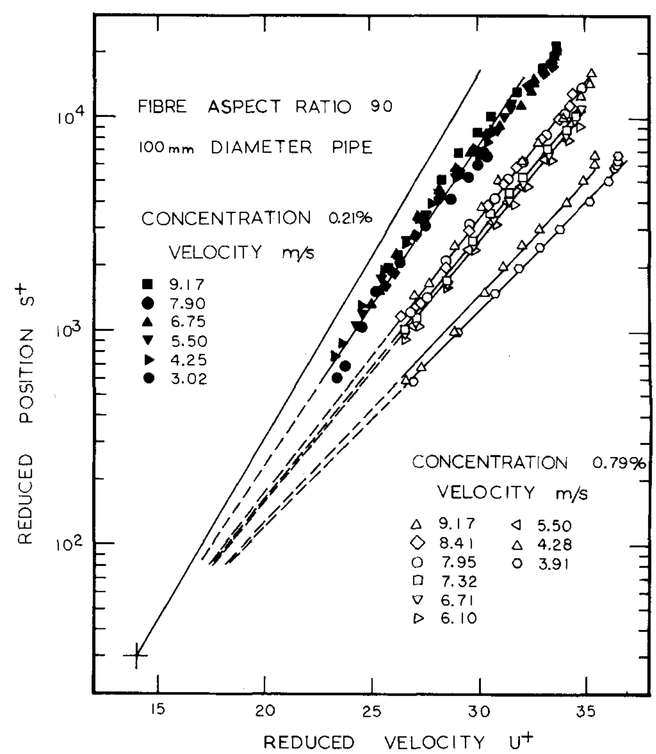

Fig. 1. Typical reduced velocity profiles for two suspensions of papermaking fibers.

The universal velocity profile for Newtonian fluids [Equation (3)] has been included in Figure 1 for reference:

$$U^+ = \frac{1}{0.4} \ln S^+ + 5.5 \tag{3}$$

Values of bulk velocity calculated by integrating velocity profiles were compared with values of bulk velocity measured directly with a calibrated, magnetic flowmeter. The average deviation of corresponding bulk velocities was 1.5%, and the individual deviations exceeded 3.0% in only two of the thirty-two profiles measured. Velocity profiles measured for the flow of pure water were in good agreement with the accepted profile [Equation (3)].

Reduced velocity profiles for fiber suspensions are approximately linear in regions of turbulent shear. The gradient of each line is less than the Newtonian value of 0.4 and is termed the apparent von Kármán constant K. The parameter K is a measure of the momentum transfer ability of a suspension and was found to be a function of fiber concentration and bulk velocity. It has been shown (Lee and Duffy, 1967*b*) that variations of K can be explained in terms of the relationship between

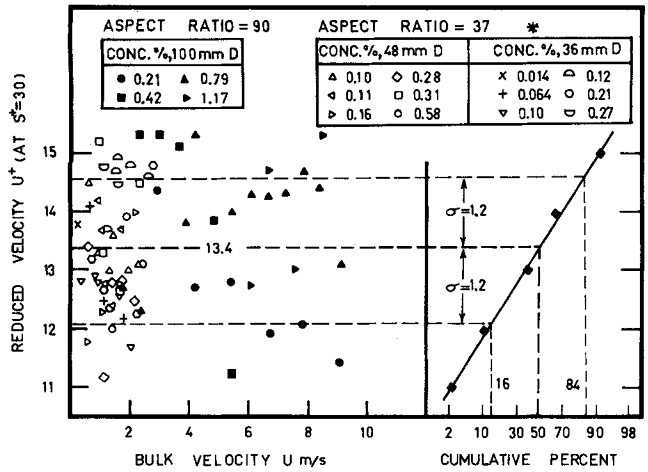

Fig. 2. Graphs of U^+ (at $S^+ = 30$) vs. bulk velocity U and cumulative frequency distribution. *Data from Seely (1968).

intensity of turbulence and the scale of fiber agglomeration in a suspension.

In fully developed turbulent flow, two regimes can be recognized from the variations of K with bulk velocity. At low turbulence intensities, values of K for a particular suspension increase with increase in bulk velocity. At high turbulence intensities, there is a regime in which K is independent of flow rate. As concentration decreases in this regime, the lower bulk velocity limit decreases and the limiting value of K increases to approach that for water.

In addition, there is a regime of partially developed turbulence at very low turbulence intensities characterized by a central plug of undisrupted fibers and a turbulent fiber/water annulus, in which the value of K varies significantly with distance from the pipe wall.

THE WALL REGION

Local mean velocities close to the pipe wall can be estimated by extrapolating linear reduced profiles from the turbulent core (dashed lines in Figure 1). All the extrapolated profiles (including the Newtonian profile) intersect near the coordinates $U^+ = 14$, $S^+ = 30$ (generally accepted as the approximate boundary condition for the Newtonian profile in the turbulent core). A value of U^+ at $S^+ = 30$ was calculated from a least-squares, best-fit line for each of the linear reduced profiles obtained in this investigation and from the data of Seely (1968). These values are plotted vs. bulk velocity in Figure 2. In the general scatter of results, there is no significant dependence of the values of U^+ at $S^+ = 30$ on fiber concentration, fiber aspect ratio, bulk velocity, or pipe diameter. Included in Figure 2 is a cumulative frequency distribution on normal probability coordinates. The linear relationship indicates that the results follow an approximately normal distribution with a mean of 13.4 and a standard deviation of 1.2. It appears that fibers in water do not greatly influence the processes of turbulence generation in the region of flow adjacent to the pipe wall. Drag reduction occurs as a result of fibers reducing momentum transfer in the turbulent core.

VELOCITY PROFILE CORRELATIONS

The general expression for a linear, reduced velocity profile that passes through the point with coordinates $U^+ = 14$, $S^+ = 30$ can be written in terms of a single parameter K:

$$U^+ = \frac{1}{K} \ln S^+ + \left[14 - \frac{3.4}{K} \right] \qquad (4)$$

Equation (4) is a convenient correlation of velocity profiles at high bulk velocities when K is a constant for a particular suspension.

At lower bulk velocities, when K increases as bulk velocity increases, it is more convenient to represent local mean velocities by

$$U^+ = \frac{1}{K} \ln y + \delta \qquad (5)$$

where δ is a suspension parameter. This is possible only because the small increase in K with increase in flow rate is fortuitously compensated by a corresponding, small increase in δ, and average values of these parameters for a particular suspension can be used in Equation (5).

RELATIONSHIPS BETWEEN FLOW RESISTANCE AND VELOCITY PROFILES

An expression for high bulk velocities U of turbulent fiber suspension flow through a pipe with diameter D [Equation (6)] was obtained by integrating local mean velocities given by Equation (4) over the turbulent core and by neglecting the very small flows in the wall region $0 < S^+ < 30$:

$$U = U^* \left[\frac{1}{K} \ln \frac{D\, U^*}{2\, \nu} + \left(14 - \frac{4.9}{K} \right) \right] \qquad (6)$$

Equation (6) can be reexpressed in terms of friction factor ϕ [defined by Equation (7)] and Reynolds number [defined by Equation (8)] to give Equation (9):

$$\phi = \frac{\tau_w}{\rho U^2} = \left(\frac{U^*}{U} \right)^2 \qquad (7)$$

$$Re = \frac{UD}{\nu} \qquad (8)$$

$$\frac{1}{\sqrt{\phi}} = \frac{1}{K} \ln Re \sqrt{\phi} + \left[14 - \frac{5.6}{K} \right] \qquad (9)$$

Equation (9) is a pipe flow resistance formula for turbulent fiber suspensions and describes gross flow solely in terms of the apparent von Kármán constant K. When K equals the Newtonian value of 0.4, the expression reduces to the Kármán-Prandtl law and for values of K less than 0.4 predicts drag reduction. Equation (9), like Equation (4), is most likely to be useful at high bulk velocities, when K is a constant for a particular suspension.

Good agreement has been obtained between values of K calculated from velocity profile data by using Equation (4) and values of K calculated from flow resistance data by using Equation (9) (Lee and Duffy, 1976*a*). The relationship between flow resistance and the apparent von Kármán constant K, given by Equation (9), is further supported by data obtained by Vaseleski and Metzner (1974) for a number of fiber suspensions and a wide range of pipe diameters. They represented data for each suspension by a single straight line on a graph of $1/\sqrt{f}$ vs. $\ln(Re \sqrt{f})$ (where f equals 2ϕ). Close agreement has been obtained between values of K calculated independently from the gradient $(1/K)$ and the corresponding intercept $[14 - (5.6/K)]$ for each of their lines (see Table 1). It should be noted that data obtained by Vaseleski and Metzner for suspensions of Turner Brothers asbestos fibers were not included because of shear degradation effects.

A flow resistance formula for fully developed turbulent fiber suspension flow at low bulk velocities [Equation (10)] was developed from Equation (5) by using a

TABLE 1. COMPARISON OF VALUES OF THE APPARENT VON KÁRMÁN CONSTANT K CALCULATED INDEPENDENTLY FROM THE GRADIENT AND INTERCEPT OF THE GRAPHICAL DATA OF VASELESKI AND METZNER (1974)

Additive fibre	Concentration, %	Apparent von Kármán constant K — K from gradient	K from intercept
Nylon	1.00	0.35	0.35
JM* Asbestos	0.01	0.34	0.36
	0.02	0.30	0.32
	0.08	0.28	0.29
	0.25	0.25	0.26
	0.50	0.11	0.11

* Johns-Manville, Asbestos, Quebec, Canada.

similar analysis to that outlined above for high bulk velocities:

$$\phi = \left[\delta - \frac{1}{K} \left(\frac{3}{2} - \ln \frac{D}{2} \right) \right]^{-2} \tag{10}$$

The salient feature of Equation (10) is the prediction that friction factor is independent of bulk velocity provided that the small increase in K with increased bulk velocity is compensated by a corresponding, small increase in δ. A regime of turbulent fiber suspension flow characterized by a value of friction factor approximately independent of bulk velocity has been observed in this and several previous investigations (for example, Mih and Parker, 1967; Seely, 1968). Good agreement has been obtained between values of ϕ calculated from Equation (10) by using average values of K and δ obtained from velocity profile measurements and values of ϕ calculated from flow resistance data by using Equation (7) (Lee and Duffy, 1976*a*).

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LITERATURE CITED

Kale, D. D., and A. B. Metzner, "Turbulent Drag Reduction in Fiber-Polymer Systems: Specificity Considerations," *AIChE J.*, **20**, 1218 (1974).

Kerekes, R. J. E., "Turbulent Drag Reduction in Pipe Flow of Ideal Fibre Suspensions," Ph.D. dissertation, McGill Univ., Canada (1970).

Lee, P. F. W., and G. G. Duffy, "An Analysis of the Drag Reducing Regime of Pulp Suspension Flow," accepted by *Tappi* (1976*a*).

———, "Velocity Profiles in the Drag Reducing Regime of Pulp Suspension Flow," submitted to *Appita* (1976*b*).

Lee, W. K., R. C. Vaseleski, and A. B. Metzner, "Turbulent Drag Reduction in Polymeric Solutions Containing Suspended Fibers," *AIChE J.*, **20**, 128 (1974).

Mih, W., and J. Parker, "Velocity Profile Measurements and a Phenomenological Description of Turbulent Fiber Suspension Flow," *Tappi*, **50**, 237 (1967).

Radkin, I., J. L. Zakin, and G. K. Patterson, "Drag Reduction in Solid-Fluid Systems," *AIChE J.*, **21**, 358 (1975).

Seely, T., "Turbulent Tube Flow of Dilute Fiber Suspensions," Ph.D. dissertation, The Institute of Paper Chemistry, Appleton, Wisc. (1968).

Vaseleski, R. C., and A. B. Metzner, "Drag Reduction in Turbulent Flow of Fiber Suspensions," *AIChE J.*, **20**, 301 (1974).

Virk, P. S., "Drag Reduction Fundamentals," *ibid.*, **21**, 625 (1975).

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Sorption of Oxygen, Nitrogen, Carbon Monoxide, Methane, and Binary Mixtures of these Gases in 5A Molecular Sieve

Equilibrium data for the sorption of oxygen, nitrogen, carbon monoxide, and methane in 5A molecular sieve are analyzed in terms of a simple theoretical model isotherm. The model provides an excellent correlation of the single-component isotherms over the entire concentration range, and it is shown that equilibrium data for sorption of binary mixtures of these gases are correctly predicted by the model using the parameters (Henry constants and molecular volumes) derived from analysis of the single-component isotherms. The model predicts that mixtures of two sorbates with equal molecular volumes should show approximately ideal solution behavior in the adsorbed phase. The experimental data of Lederman for the sorption of nitrogen-methane mixtures show the expected behavior over a wide range of pressures.

DOUGLAS M. RUTHVEN

Department of Chemical Engineering
University of New Brunswick
Fredericton, N.B., Canada

SCOPE

Industrial adsorption separation processes generally involve the sorption of multicomponent mixtures, and a reliable predictive method of estimating mixture equilibria from single-component isotherm data is required for the proper design and modeling of such processes. Of the available methods which have so far been suggested, the method of Myers and Prausnitz (1965), which is based on the assumption of ideal behavior in the adsorbed phase,